Art-Inspired Modeling and Visualization

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The mathematical investigations of Roger Penrose inspired Dutch artist M.C. Escher to illustrate in his drawings the constructive elements of this world. We introduce unusual higher-dimensional computer reconstructions of some drawings for presenting our research results in solving geometric modeling problems arising from analysis of Escher's artworks. The following identified problems had no general solutions in geometric modeling: the metamorphosis between 2D arbitrary polygons similar to shape transformations in "Day & Night" (1938), the simulation of 3D shape reconstruction from its 2D projection as it happens in "Reptiles" (1943), and the surface trimming depicted in the "Rind" (1955) and other Escher's drawings. We applied the Function Representation (FRep) of geometric shapes by continuous functions of point coordinates [1, 4] to solve these problems.

A visually smooth transformation (or metamorphosis) between two arbitrary shapes can find a general solution if both shapes are represented by real functions and the transformation is described as an interpolation between these functions. First, we represent a 2D polygon by a real function of point coordinates taking zero value at polygon edges. An arbitrary polygon can be represented by a set-theoretic expression involving union and intersection operations on the half-planes passing through the polygon edges. The formula for the function defining a polygon can be obtained from the set-theoretic expression by replacing each half-plane by its defining function while set-theoretic operations are replaced by the corresponding R-functions, see details in [2].

We implemented the above polygon-to-function conversion approach and applied it to several geometric modeling problems including shape metamorphosis and reconstruction from projections. The original Escher's drawing and our reconstruction are shown in Fig. 1. Both polygons, the bird shape and the rectangle, are represented by corresponding real functions. The linear interpolation between them determines the in-between shapes.

Figure 1: Shape metamorphosis:
a) the original transformation (fragment of "Day & Night", ©2009 The M.C. Escher Company B.V., the Netherlands);
b) a polygonal shape transformation using interpolation between defining functions.

The problem of 3D shape reconstruction from its projection is visually stated in the "Reptiles" drawing by M.C. Escher (Fig. 2a), where 3D reptiles emerge into space from their 2D images. To automate such a reconstruction we employ the described above functionally represented polygons as illustrated in Fig. 2b-d.

For a 2D polygon outlining the projection, a height field is generated by taking only the positive values of the defining function for the polygon. This height field represents a relief that can be attached to any support object. We apply the offsetting along the normal operation [1] (with the offset distance modulated by the height field), which can be considered one of techniques of computer-aided relief carving [2].

The next part of our work was inspired by several drawings of M.C. Escher, namely "Sphere Spirals" (1958) shown in Fig. 3a, "Bond of Union" (1956), and "Rind" (1955), showing spiral shaped surface sheets cut out of a sphere and human head surfaces.

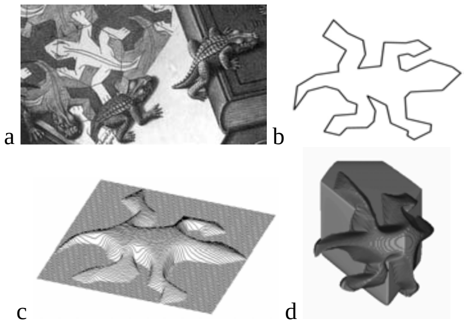

Figure 2: Reconstruction from projection:
a) a fragment of Escher's "Reptiles" (©2009 The M.C. Escher Company B.V., the Netherlands);
b) an initial concave polygon as a reptile boundary;
c) a height field inside the polygon generated using the polygon-to-function conversion procedure;
d) a reptile carved on a block using offsetting controlled by the depth data.

These art works raised the question on the ways one can define and visualize a geometric model for a surface patch of this type. Modeling and visualization of the spiral type surface patches of complex shapes using parametric surfaces seems to be a difficult task. An alternative is to use implicit surfaces.

Figure 3: Trimming a spherical surface:
a) "Sphere Spirals" (©2009 The M.C. Escher Company B.V., the Netherlands);
b) a trimming solid;
c) a trimmed sphere surface.

A surface patch can be represented as a set-theoretic difference between some initial implicit surface and a trimming solid. For modeling Escher's "Sphere Spirals" a trimming solid consisting of a union of three spirals can be introduced as shown in Fig. 3b. We applied for rendering purposes a special algorithm for polygonization of trimmed implicit surface [3]. Fig. 4 was produced after Escher's "Rind" (1955) where the initial FRep solid (human head model) was constructed using set operations with R-functions on algebraic primitives, soft objects, and convolution solids. Then, its surface was trimmed using the same trimming solid as in the previous example.

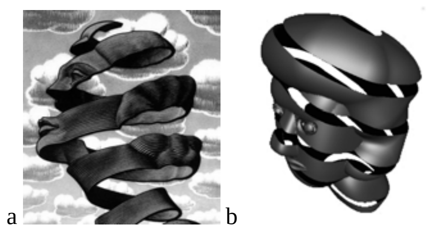

Figure 4: Trimming complex surface of an FRep object:
a) a fragment of Escher's "Rind" (©2009 The M.C. Escher Company B.V., the Netherlands);
b) an implicit surface of a head trimmed by a spiral solid.

In this work, we tried to reproduce 2D drawings and to reconstruct from them models ascending in spaces of higher dimensions. The proposed algorithms, developed software tools and corresponding experiments can be considered as related to digital preservation, further usage and interpretation of cultural heritage. One of the interesting forms of reconstruction of artistś images in 3D space is digital fabrication of his works, which is the future direction of our research.

References

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