Topological Methods for Motion Data Analysis

Mykhaylo Nykolaychuk, Klaus Richter, Christian Rössl, Holger Theisel

Fraunhofer IFF Magdeburg, Otto-von-Guericke-Universität Magdeburg
Email: {mykhaylo.nykolaychuk,klaus.richter}@iff.fraunhofer.de
{roessl,theisel}@isg.cs.uni-magdeburg.de

.

1 Introduction

We present a new method for analyzing image-based motion data with topological methods for flow fields. A video sensor captures motion of subjects and yields discrete samples of velocity vectors in the image domain. We show how to construct a smooth, continuous, globally defined bidirectional flow field which evaluates determined vectors at given samples. We then apply a topological analysis of this continuous flow field which yields a segmentation into regions of similar flow behavior, i.e., regions of similar motion.

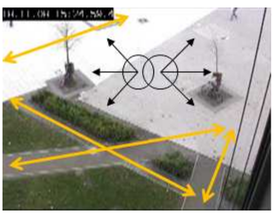

Figure 1: Motion scene

This segmentation can be used for further analysis like detecting atypical motions. We tested our method empirically and provide results for two different scenarios (see Figure 1 for one scenario).

2 Setup

We use a video camera in combination with optical flow to extract motion data from a video sequence for traffic applications (see [2] for details). In our setup the sensor is fixed and captures a certain region. For any detected motion of any subject we are provided with a sequence of a subject's id, its positions in image-space and a time-stamp. From these data we construct trajectories of individual subjects in image space. The inital trajectories are smoothed in oder to suppress artifacts disturbing effects like distortion, overlapping, occlusion, or fusion of moving objects. The tangents of the

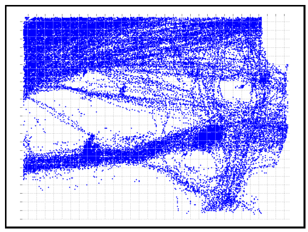

Figure 2: Raw motion data

trajectories are sampled on a regular grid. This provides us a configuration of vector samples scattered in the domain (see Figure 2), which constitue the input to our method.

3 Vector field generation

We need to generate a suitable continuous parametric model from the given discrete set of vector samples. A standard approach is least-squares approximation of a bivariate function, e.g., tensor-product B-spline. Unfortunately, this approach does not work in our setting: our goal is fitting a *bidirectional* vector field (or a tensor field), i.e., we want to accurately fit the direction of velocity vectors but not their orientation. This *orientation-invariance* cannot be modeled by a linear least-squares fit. We provide a different formulation of the problem which leads to solving an *eigenvalue problem*.

Energy minimization Let $\mathbf{v}_i \in \mathbb{R}^2$ denote the given vector samples at positions (x_i, y_i), and

$\mathbf{w}(x, y)$ denotes the unknown parametric represention of the vector filed in any suitable basis. We are only interested in matching the direction of vectors and disregard both, orientation and magnitude. Therefore, instead of $\mathbf{w}$ and $\mathbf{v}$ being parallel, we require that $\mathbf{w}(x_i, y_i)$ as *orthogonal* to $\mathbf{v}_i^\perp =: (-v_i, u_i)^\top$ as possible, where $(\cdot)^\perp$ denotes a rotation by $\frac{\pi}{2}$. For the best fitting bidirectional vector field we have

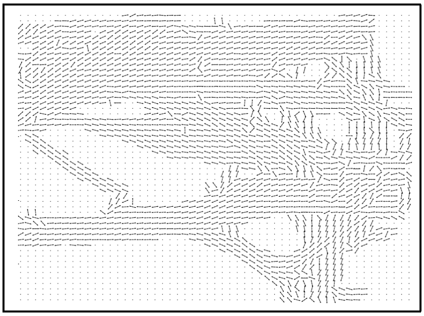

Figure 3: Estimated vector field

$$E = \sum_{i=1}^{m} \left(\mathbf{w}(x_i, y_i)^\top \mathbf{v}_i^\perp \right)^2 \rightarrow \min \ ,$$

i.e., we penalize non-orthogonality (see Figure 3). The minimization of the above functional leads to solving an eigenvalue problem.

Regularization and velocity bias Parts of the domain do not contain any vector samples. The approximation method described above may not be able to "interpolate" meaningful vectors across such regions. The reason is that, e.g., for B-splines, the Schoenberg-Whitney conditions are generally not satisfied, and the arising system matrix is hence singular. (Note that the eigenvalue problems yields a solution though, however, it yields vectors of very small magnitude in "unknown" regions.) The standard approach to solve this problem is adding a regularization term, i.e., taking into account additional constraints usually on smoothness. We penalize the norm of first order partials.

The formulation as is effectively yields a weighted least-squares approximation (our energy include not only angles but also magnitudes of samples which implicitly provides a weighting of samples). For our purpose, such weighting is neither meaningful nor does it provide better results, hence we normalize all samples. Note that recovering speed as a scalar value would simply constitute a second step resulting in least-squares fitting a scalar field.

4 Topology extraction

We extract a topological skeleton of the smooth bidirectional flow field $\mathbf{w}$ which provides the segmentation of the image domain into distinct regions of similar motion. We have to find first critical points and second integrate streamlines starting from critical points to construct separatrices and hence segment boundaries. A bidirectional flow is essentially a 2nd order symmetric *tensor field* $\mathbf{T}(x, y)$, criritcal points are degenerate points (x_c, y_c) of the tensor field $\mathbf{T}$ for which the eigenvalues of $\mathbf{T}(x_c, y_c)$ are equal, and they can be classified into *wedges* or *trisectors* based on certain partials [1].

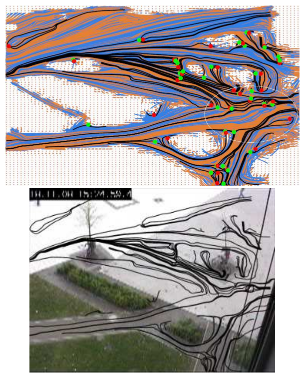

Figure 4: Topology of persons motion trajectories

We use a 4th order Runge-Kutta method for numerical integration on $\mathbf{w}$ and modify the standard algorithm such that the orientation is kept consistent during integration.

Our experimental results confirm the effectiveness of our method (see Figure 4).

References

[1] T. Delmarcelle and L. Hesselink. The topology of symmetric, second-order tensor fields. In *IEEE Visualization*, pages 140–147, 1994.

[2] V. Kastrinaki, M. Zervakis, and K. Kalaitzakis. A survey of video processing techniques for traffic applications. *Image and Vision Computing*, 21:359–381, 2003.

Acknowledgement The Autors are funded by the German Ministry of Eductaion and Science (BMBF) within the ViERforES project (no. 01IM08003C).